Learning About Non-Newtonian Fluids in a Student-Driven Classroom

D. R. Dounas-Frazer, J. Lynn, A. M. Zaniewski, and N. Roth, University of California at Berkeley, Berkeley, CA

We describe a simple, low-cost experiment and corresponding pedagogical strategies for studying fluids whose viscosities depend on shear rate, referred to as "non-Newtonian fluids." We developed these materials teaching for the Compass Project,[1] an organization that fosters a creative, diverse, and collaborative community of science students at UC Berkeley. Incoming freshmen worked together in a week-long residential program to explore physical phenomena through a combination of conceptual model-building and hands-on experimentation. During the program, students were exposed to three major aspects of scientific discovery: developing a model, testing the model, and investigating deviations from the model.

We chose to study non-Newtonian fluids because they are not only the subject of active research,[2] but they are also ubiquitous in our everyday lives; examples include ketchup, toothpaste, and sand. Despite its importance, fluid dynamics is a subject that is typically neglected in high school physics curricula. Nevertheless, non-Newtonian fluids are amenable to a wide range of engaging, collaborative activities.[3]

One of our goals was for students to derive Stokes' law, which describes the velocity dependence of the drag force experienced by a ball moving in a viscous fluid. Stokes' law, which is valid when the flow is sufficiently viscous that no turbulence develops, is:[4]

$$F = 6\pi C R \eta v, \tag{1}$$

where R and v are the ball's radius and speed, respectively, η is the fluid's dynamic viscosity, and C is a dimensionless constant that depends on the geometry of the fluid's container ($C = 1$ when the container is very large compared to the size of the ball).[5] Fluids for which η is independent of v are known as Newtonian, whereas those for which it is not are non-Newtonian. Thus, for Newtonian fluids, Stokes' law predicts a linear relationship between the speed of the ball and the drag force it experiences. Non-Newtonian fluids, on the other hand, deviate from Stokes' law, often by exhibiting a nonlinear dependence of v on F. We emphasize that this distinction is only valid when the Reynolds number is low and the flow is laminar. At high Reynolds numbers, turbulence invalidates the linear relationship between v and F for Newtonian fluids.[4]

We discuss how our students derived Eq. (1) and describe an experiment that they used to measure $v(F)$ for two different fluids: corn syrup (Newtonian) and a mixture of cornstarch and water called "oobleck" (non-Newtonian). The apparatus we present is different from others[6-9] that discriminate between Newtonian and non-Newtonian fluids in that it is relatively simple and low cost. In contrast to falling-ball viscometers,[10] students do not need to consider buoyant forces to understand the operation of our apparatus. Finally, students examined oobleck under a microscope, thereby illustrating the origins of oobleck's non-Newtonian behavior. Over the course of the week, students spent a total of 10 hours in the classroom engaging in small and large group discussions, three hours performing experiments, and about six hours completing collaborative homework assignments; we present a somewhat streamlined account of this work.

Deriving Stokes' law

To guide the students' derivation of Stokes' law, we followed a classroom model called Modeling Discourse Management (MDM),[11] and incorporated the method Think-Pair-Share.[12] Students worked in groups of four, using a small whiteboard to record their work and share ideas with the class. MDM prescribes that if the class is missing a critical or interesting idea in their small group discussions, teachers should attempt to "seed" discussions by asking simpler related questions that could serve as stepping stones to the main result.

We introduced the concept of viscosity by asking students to describe what happens to a thick fluid between two parallel plates separated by distance Δy when one plate moves at constant speed Δv relative to the other, fixed plate. An example of a question we used to seed discussion in one small group was, "What is the fluid right next to the plate doing?" Students correctly intuited a flow pattern with a position-dependent velocity that increases smoothly from zero at the stationary plate to Δv at the moving plate, a pattern commonly referred to as "Couette flow,"[4] and that there must be forces exerted both between neighboring layers of fluid and at the plate-fluid interface. Additionally, students guessed that processes at the molecular level must be responsible for this "sticking" or "dragging." We took the standard approach of lumping all of the microscopic details into one macroscopic quantity, the viscosity η.

We asked students to identify the parameters on which the force exerted on the top plate depends, and to articulate how the force depends on those factors. Students identified four relevant dependencies, namely: F increases with increasing η, Δv, and plate area A, and F decreases with increasing Δy. We then asked students to construct a quantitative expression for the drag force on one fluid layer due to a neighboring layer using dimensional analysis, a technique that has been used to study other aspects of fluid flow.[13] Given that viscosity has units of Pa·s, students successfully constructed the only simple expression that satisfies the above dependencies:
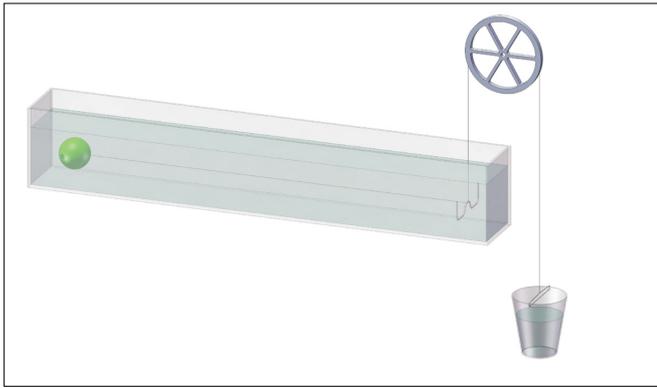
Fig. 1. Schematic of apparatus used to measure velocity dependence of fluid drag forces.

Fig. 2. Student experimentalists measuring drag forces in oobleck.

$$F = \eta A \Delta v / \Delta y, \qquad (2)$$

which reduces to Newton's law of viscosity in the limit $\Delta v / \Delta y \to dv/dy$.[4]

Next, we asked the students to use Eq. (2) to make a prediction about the drag force on a ball of radius R moving at velocity v through an otherwise stationary fluid that is infinite in extent. Students made the following approximations (assisted by seed questions as necessary): the area over which the force is applied is approximately the surface area of the ball, so $A \approx 4\pi R^2$; because the fluid far from the ball is stationary whereas the fluid near the ball moves with speed v, the change in velocity is $\Delta v = v$; and, since R is the only available length scale, the characteristic distance over which the change in velocity occurs is $\Delta y \approx R$. These approximations yield

$$F \approx 4\pi R \eta v, \qquad (3)$$

consistent with Stokes' law. Because determination of C in Eq. (1) was beyond the scope of our goals, we focused on the scaling of F with v rather than the absolute magnitude of the force.

Apparatus and procedure

The apparatus, shown in Fig. 1, consists of a Ping-Pong ball submerged in a fluid-filled acrylic trough (60 cm long, 8 cm wide, and 10 cm tall), with a wire that connects the ball to a counterweight of mass m via a crossbeam and pulley. When the weight is released from rest, the ball accelerates to a constant (terminal) velocity v, at which point the counterweight exactly balances the drag force experienced by the ball: $F = mg$, where $g = 9.8$ m/s^2 is the standard gravity on Earth. Because the ball and counterweight are attached by a taut wire, they travel at the same speed. Thus F and v can be determined by measuring the mass and speed of the counterweight. The counterweight was a 3-g cup filled with up to 60 mL of water, which allowed for an easily scalable system.

Three holes were drilled in the ball, a small hole of diameter 3 mm and two large ones of 6 mm. The purpose of the holes was twofold: first, to facilitate attachment of the ball to the wire, and second, to allow the ball to fill with fluid in
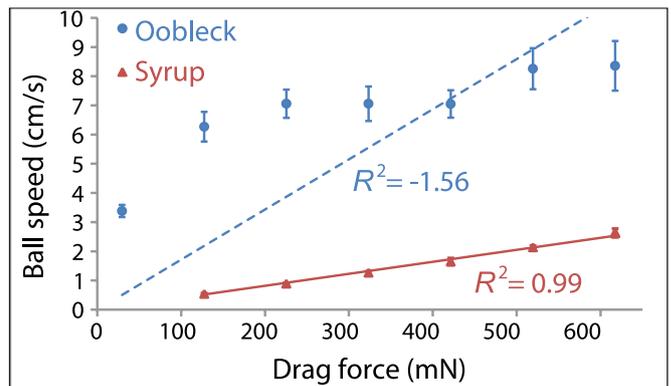

Fig. 3. Weighted class average of student measurements of ball speed v as a function of drag force F for oobleck (blue circles) and corn syrup (red triangles). The function $v(F) = \alpha F$ was fit to each data set. Fitted curves are plotted as dashed blue and solid red lines for oobleck ($\alpha = 0.2$ s/kg) and syrup ($\alpha = 0.04$ s/kg), respectively. The nonlinear dependence of v on F in oobleck indicates that it is a non-Newtonian fluid.

order to prevent it from floating or sinking. The ball was attached to the wire by threading the wire through the small hole and tying it to a plastic bead 5 mm in diameter. Though only one large hole was needed to thread the bead, the second hole greatly reduced the time needed to fill the ball with the ambient fluid.

Although we, the teachers, designed and built the apparatus, students developed their own measurement procedure. The class reached consensus on the procedure, and all groups followed the same steps. To measure v, students marked four successive 10-cm intervals on the wire with flags of tape (Fig. 2), and timed them with a stopwatch as they passed the bottom edge of the trough. For a given counterweight mass, the experiment was repeated four times. In all their experiments, students allowed the ball to travel an initial distance of 10 cm before measuring its speed to ensure that it had reached terminal velocity. This distance was determined empirically.

Results and discussion

The students fit the $v(F) = \alpha F$ model to their data using Microsoft Excel, and the corresponding correlation coefficient R^2 was used to determine whether or not a fluid was Newtonian: $R^2 \approx 1$ for Newtonian fluids, and $R^2 \ne 1$ indicates
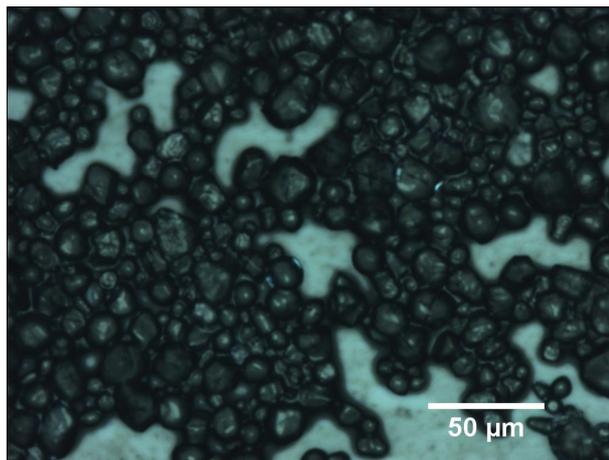
Fig. 4. Microscopic image of oobleck. The best images were obtained by looking near the edges of a droplet or through a very thin sample.

non-Newtonian behavior. For Newtonian fluids, the viscosity could in principle be determined from the slope α using Stokes' law. To do so, the constant C appearing in Eq. (1) must be determined for the particular apparatus.[5,7] However, the goal of our experiment was to test the prediction of linear relationship between v and F, thereby discriminating between Newtonian and non-Newtonian fluids. For this purpose, it is sufficient to determine whether the data are linear. The results of the students' experiments, shown in Fig. 3, indicate that syrup is Newtonian ($R^2 = 0.99$) whereas oobleck ($R^2 = -1.56$) is not.

In a subsequent class, we encouraged students to think about microscopic differences between oobleck and corn syrup that could result in their macroscopically different behavior. After forming hypotheses of what they expected to see, students examined a thin sample of oobleck under a microscope. They observed that corn starch crystals were not dissolved but suspended in the water, as can be seen in Fig. 4. Packing and jamming of the crystals, two mechanisms for the shear-thickening nature of suspensions,[2] were observed by agitating the sample.

In conclusion, students developed a conceptual and formal understanding of fluid viscosity, measured viscous forces of various fluids, and explored the microscopic origins of oobleck's non-Newtonian behavior. In addition to identifying non-Newtonian fluids, the apparatus we designed has other potential uses, e.g., determination of the velocity dependence of turbulent drag forces or viscometry. The pedagogical and experimental methods described here are appropriate for advanced high school and undergraduate students.

References
1. N. Roth, P. Gandhi, G. Lee, and J. Corbo, "The Compass Project: Charting a New Course in Physics Education," arXiv:1211.4893, Submitted to *Physics Today Points of View* (Oct. 2012).

    B. F. Albanna, J. C. Corbo, D. R. Dounas-Frazer, A. Little, and A. M. Zaniewski, "Building Classroom and Organizational Structure Around Positive Cultural Values," arXiv:1207.6848, Submitted to 2012 *PERC Proceedings* (Aug. 2012).

    See www.berkeleycompassproject.org.

2. X. Cheng, J. H. McCoy, J. N. Israelachvili, and I. Cohen, "Imaging the Microscopic Structure of Shear Thinning and Thickening Colloidal Suspensions," *Sci.* **333**, 1276–1279 (Sept. 2011).

3. P. Habdas, E. R. Weeks, and D. G. Lynn, "Squishy materials," *Phys. Teach.* **44**, 276–279 (May 2006).

4. G. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University Press, 1967).

5. A. Ambari, B. Gauthier-Manuel, and E. Guyon, "Direct measurement of tube wall effect on the Stokes force," *Phys. Fluids* **28**, 1559–1561 (Jan. 1985).

6. T. Seckin and S. M. Kormaly, "An easy-to-build rotational viscometer with digital readout," *J. Chem. Educ.* **73**, 193–194 (Feb. 1996).

7. M. Dolz, J. Delegido, A. Casanovas, and M.-J. Hernandez, "A low-cost experiment on Newtonian and non-Newtonian fluids," *J. Chem. Educ.* **82**, 445–447 (March 2005).

8. L. Courbin, G. Cristobal, M. Winckert, and P. Panizza, "Design of a low cost Zimm–Crothers viscometer: From theory to experiment," *Am. J. Phys.* **73**, 851–855 (Sept. 2005).

9. R. M. Digilov, B. Gurfinkel, and M. Reiner, "Analysis of non-Newtonian power-law liquids with a weight-controlled capillary viscometer," *Am. J. Phys.* **76**, 968–974 (Oct. 2008).

10. M. Brizard, M. Megharfi, E. Mah'e, and C. Verdier, "Design of a high precision falling-ball viscometer," *Rev. Sci. Instrum.* **76**, 025109 (Jan. 2005).

11. D. M. Desbien, *Modeling Discourse Management Compared to Other Classroom Management Styles in University Physics,* PhD thesis, Arizona State University, 2002.

12. F. Lyman, in *Mainstreaming Digest: A Collection of Faculty and Student Papers*, edited by A. S. Anderson (The University of Maryland, 1981).

13. D. Guerra, K. Corley, P. Giacometti, E. Holland, M. Humphreys, and M. Nicotera, "An introduction to dimensionless parameters in the study of viscous fluid flows," *Phys. Teach.* **49**, 175–179 (March 2011).

D. R. Dounas-Frazer, Department of Physics, University of California at Berkeley, Berkeley, CA 94720-7300; drdf@berkeley.edu

J. Lynn, Department of Physics and Theoretical Astrophysics Center, UC Berkeley

A. M. Zaniewski, Department of Physics, UC Berkeley, Materials Science Division, Lawrence Berkeley National Lab, Berkeley, CA 94720; and Center of Integrated Nanomechanical Systems

N. Roth, Department of Physics and Theoretical Astrophysics Center, UC Berkeley